Directions: Answer the following question(s).

1 Mark is asked to find the maximum value of the function shown.

$$f(x) = -x^2 + 4x + 4$$

He decides to complete the square to reveal the maximum value. Which shows the function Mark created, and the maximum value of this function?

A. $f(x) = (x - 2)^2 + 4$, and the maximum value of $f(x)$ is 4.

B. $f(x) = -(x - 2)^2$, and the maximum value of $f(x)$ is 0.

C. $f(x) = -(x - 2)^2 + 8$, and the maximum value of $f(x)$ is 8.

D. $f(x) = -(x - 2)^2 - 4$, and the maximum value of $f(x)$ is –4.

Master ID: 307985 Revision: 5

Correct: C

Rationale:

A. This is the result of completing the square to get the function into the form $-(x - h)^2 + k$, but the constant term (4) is left out in the first step.

B. This is the result of completing the square to get the function into the form $-(x - h)^2 + k$, but the squared term (4) is subtracted instead of added.

C. This is found by solving $f(x) = -(x^2 - 4x) + 4 \rightarrow f(x) = -(x - 2)^2 + 4 + 4 \rightarrow f(x) = -(x - 2)^2 + 8$. The vertex of the parabola opening down is (2, 8), making the maximum 8.

D. This is the result of completing the square to get the function into the form $-(x - h)^2 + k$, but the constant term (4) is left out, and the value needed to complete the square (4) is subtracted instead of added.

Standards:

CCSS.Math.Content.HSA-SSE.B.3.b

2 What is the maximum value of the function?

$$g(y) = -y^2 + 12y + 45$$

A. 81

B. 9

C. 189

D. –99

Master ID: 307982 Revision: 4

Correct: A

Rationale:

A. Convert the equation to vertex form by completing the square. Rearrange the equation as $g(y) = -(y^2 - 12y) + 45$, and then complete the square: $g(y) = -(y^2 - 12y + 36) + 45 + 36 \rightarrow g(y) = -(y - 6)^2 + 81$. This is the standard vertex form of the equation. It shows that a maximum of $g(y)$ = 81 occurs where $y = 6$.

B. This results from subtracting the term $(12/2)^2$ from 45 instead of adding it.

C. This is the result of squaring 12 (the coefficient of the y term) instead of 12/2, before adding it to 45.

D. This is the result of squaring 12 (the coefficient of the y term) instead of 12/2, and then subtracting that from 45.

Standards:

CCSS.Math.Content.HSA-SSE.B.3.b

3 Enter an equation for the line of symmetry for the function defined by $f(x) = 4x^2 + 8x + 3$.

Use the on-screen keyboard to type your answer in the box below.

Web Only Interaction

Master ID: 2473300 Revision: 1

Rubric: 1 Point(s)

The line of symmetry is **x = –1.**

This line is a vertical line through the vertex, found as follows:

$$x = \frac{-b}{2a} = \frac{-8}{8} = -1$$

Standards:

CCSS.Math.Content.HSF-IF.C.8

CCSS.Math.Content.HSF-IF.C.8.a

Continue: Turn to the next page.

Directions: Answer the following question(s).

4 April completed the square to find the minimum value of the function $f(x) = x^2 + 6x + 7$. Which value did she place in the blank?

$$f(x) = (x + 3)^2 + 7 + (\_)$$

A. 9
B. –9
C. –3
D. 3

Master ID: 307981 Revision: 4
Correct: B
Rationale:
A. This results from adding $\left(\frac{b}{2}\right)^2$ to the equation twice instead of adding and subtracting it.
B. To complete the square when the coefficient of the x^2 term is 1, add the quantity $(b/2)^2$ to make a perfect square, and then subtract it to preserve the equality. The solution is $f(x) = x^2 + 6x + 9 + 7 - 9 \rightarrow f(x) = (x + 3)^2 + 7 - 9$. The 9 is subtracted as the 9 was added.
C. This results from adding and subtracting b instead of $\left(\frac{b}{2}\right)^2$.
D. This results from adding b to the equation twice instead of adding and subtracting $\left(\frac{b}{2}\right)^2$.
Standards:
CCSS.Math.Content.HSA-SSE.B.3.b

5 Sandra wants to find the point on the graph where the minimum value of this equation occurs.

$$y = x^2 - 6x + 8$$

She completes the square to find the minimum value. Which function is equivalent to the original function, and at what point does the minimum value occur?

A. $y = (x - 3)^2 - 1$, with the minimum at $(3, -1)$
B. $y = (x - 3)^2 - 1$, with the minimum at $(-3, -1)$
C. $y = (x - 3)^2 + 17$, with the minimum at $(-3, 17)$
D. $y = (x - 3)^2 + 17$, with the minimum at $(3, -17)$

Master ID: 307978 Revision: 4
Correct: A
Rationale:
A. This is the result of completing the square to put the function in the vertex form $y = (x - h)^2 + k$. In this form, the vertex of the quadratic maximum or minimum corresponds to (h, k). To complete the square, add and subtract the term $(6/2)^2$ to the function as follows: $y = x^2 - 6x + 9 + 8 - 9 \rightarrow y = (x - 3)^2 + 8 - 9 \rightarrow y = (x - 3)^2 - 1$. The point of the minimum value is (3, –1).
B. This results from correctly completing the square, but using $-h$ rather than h from the vertex form $y = (x - h)^2 + k$. The minimum value occurs at the point (h, k).
C. This results from completing the square incorrectly, adding a 9 rather than subtracting it. The minimum point is also incorrect.
D. This results from completing the square incorrectly, adding a 9 rather than subtracting it.
Standards:
CCSS.Math.Content.HSA-SSE.B.3.b

Directions: Answer the following question(s).

6 The temperature, in Celsius, of a certain substance during a chemistry experiment at time t minutes is modeled by the expression below.

$$t^2 - 15t + 54$$

Which expression is equivalent to the above expression and BEST reveals the minimum temperature reached by the substance?

A. $(t + 6)(t + 9)$

B. $(t - 7.5)^2 + 2.25$

C. $(t - 7.5)^2 - 2.25$

D. $(t - 6)(t - 9)$

Master ID: 2114850 Revision: 3

Correct: C

Rationale:

A. This is the result of making sign errors in the factored form of the given expression.

B. This is the result of making a sign error in the constant term outside the parentheses when completing the square. Although this expression does reveal the 7.5, it is not equivalent to the given expression.

C. $t^2 - 15t + 54 = t^2 - 15t + (-15/2)^2 + 54 - (-15/2)^2 = (t - 7.5)^2 + 54 - 56.25 = (t - 7.5)^2 - 2.25$, which clearly reveals that the minimum temperature reached is –2.25 °C.

D. This is the result of selecting an expression that is equivalent to the given expression but that does not best reveal the minimum temperature reached.

Standards:

CCSS.Math.Content.HSA-SSE.B.3.b

7 Enter an equation for the line of symmetry for the function defined by $f(x) = 2x^2 - 20x - 7$.

Use the on-screen keyboard to type your answer in the box below.

Web Only Interaction

Master ID: 2473299 Revision: 1

Rubric: 1 Point(s)

The line of symmetry is ***x* = 5.**

This line is a vertical line through the vertex, found as follows:

$$x = \frac{-b}{2a} = \frac{20}{4} = 5$$

Standards:

CCSS.Math.Content.HSF-IF.C.8

CCSS.Math.Content.HSF-IF.C.8.a