Directions: Answer the following question(s).

1 Which of these represents $y = 2x^2 - 16x + 40$ in vertex form as well as the vertex of the function?

A. $y = 2(x - 4)^2 + 8$ with a vertex of $(4, 8)$

B. $y = 2(x + 4)^2 + 8$ with a vertex of $(-4, 8)$

C. $y = 2(x + 4)^2 - 8$ with a vertex of $(-4, -8)$

D. $y = 2(x - 4)^2 - 8$ with a vertex of $(4, -8)$

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Correct: A

Rationale:

A. This is the result of first completing the square in $y = 2x^2 – 16x + 40$ by first factoring 2 from the first two terms on the right side of the equation to give $y = 2(x^2 – 8x) + 40$. Half of –8 is then taken and squared to yield 16. This value is then added inside of the parentheses (to complete the square) and its opposite is multiplied by a factor of 2 and added to 40 outside the parentheses to obtain $y = 2(x^2 – 8x + 16) + 40 +2(–16)$. This simplifies to $y = 2(x^2 – 8x + 16) + 8$ or $y = 2(x^2 – 4) + 8$. A quadratic function in the form $y = a(x – h)^2 + k$ has a vertex at (h, k); therefore, this function has a vertex at (4, 8).

B. This is the result of first completing the square in $y = 2x^2 – 16x + 40$ by first factoring 2 from the first two terms on the right side of the equation to give $y = 2(x^2 – 8x) + 40$. Half of –8 is then taken and squared to yield 16. This value is then added inside the parentheses (to complete the square) and its opposite is multiplied by a factor of 2 and added to 40 outside the parentheses to obtain $y = 2(x^2 – 8x + 16) + 40 +2(–16)$. This simplifies to $y = 2(x^2 – 8x + 16) + 8$, which is incorrectly factored to $y = 2(x^2 – 4) + 8$. A quadratic function in the form $y = a(x – h)^2 + k$ has a vertex at (h, k); therefore, this function has a vertex at (–4, 8).

C. This is the result of incorrectly completing the square to yield $y = 2(x^2 + 4) – 8$. A quadratic function in the form $y = a(x – h)^2 + k$ has a vertex at (h, k); therefore, this function has a vertex at (–4, –8).

D. This is the result of incorrectly completing the square to yield $y = 2(x^2 – 4) – 8$. A quadratic function in the form $y = a(x – h)^2 + k$ has a vertex at (h, k); therefore, this function has a vertex at (4, –8).

Standards:

CCSS.Math.Content.HSF-IF.C.8
CCSS.Math.Content.HSF-IF.C.8.a

Continue: Turn to the next page.

Directions: Answer the following question(s).

2 A rock on Mars moves according to the equation $y = -6x^2 + 48x$, where x = time in seconds and y = height in feet.

Write the equation in vertex form $y = a(x - h)^2 + k$. Explain what information about the situation is given by the equation when it is in vertex form.

Type your answer in the box below.

Web Only Interaction

Master ID:	2115629 Revision:	4
Rubric:	1 Point(s)	

See Rubric.

2 The response is correct and complete. A level 2 response is characterized by:

- The correct vertex form of the equation; and
- A correct explanation about the meaning of the vertex.

Sample Correct Answer:

To write the equation in vertex form, factor out the –6 and then complete the square, as follows: $y = –6(x^2 – 8x) \rightarrow y = –6((x^2 – 8x) + 16) + 6(16) \rightarrow y = –6(x – 4)^2 + 96$. This form of the equation shows that the rock reaches a maximum height of 96 feet when $x = 4$ seconds.

1 The response is partially correct. A level 1 response is characterized by:

- The correct vertex form of the equation, without a correct explanation; or
- An incorrect vertex form of the equation, with an explanation about the meaning of the vertex that is otherwise correct.

0 The response is completely incorrect, there is no response, or the response is off topic.

Standards:

CCSS.Math.Content.HSF-IF.C.8

3 A. The height in feet, $h(t)$, of a small toy rocket t seconds after being fired from the top of a building is given by $h(t) = –16t^2 + 84t + 130$. Explain how to write $h(t)$ in vertex form by completing the square in order to find the maximum height attained by the rocket and the time at which this height is reached. Show your work.

B. Explain how to write $h(t)$ in factored form in order to easily determine the time at which the toy rocket hits the ground after being fired. Show your work, and show how to calculate the time.

Continue: Turn to the next page.

Directions: Answer the following question(s).

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Rubric: 4 Point(s)

4 The response demonstrates a high level of understanding. A level 4 response is characterized by:

- A correct answer in part A, as follows: The equation can be rewritten in vertex form as $h(t)$ = –16(t – (21/8))^2 + (961/4) OR $h(t)$ = –16(t – 2.625)^2 + 240.25; Time at which maximum height is reached is 2.625 seconds, and the maximum height is 240.25 feet;
- A correct explanation and work in part A, similar to "Take $h(t)$ = –16t^2 + 84t + 130 and convert it to vertex form by completing the square. To complete the square, first factor the leading coefficient of –16 from the first two terms to give $h(t)$ = –16(t^2 – (21/4)t) + 130. Then, take half of the coefficient of the t-term within the parentheses and then square it to obtain (–(21/4)/2)^2 = 441/64. This value should be added within the parentheses and then subtracted from 130 outside the parentheses once it is multiplied by –16. This step yields $h(t)$ = –16(t^2 – (21/4)t + (441/64)) + 130 – 16(–441/64) = –16(t^2 – (21/4)t + (441/64)) + 130 + (441/4) or $h(t)$ = –16(t^2 – (21/4)t + (441/64)) + (961/4). Further factoring gives $h(t)$ = –16(t – (21/8))^2 + (961/4) OR $h(t)$ = –16(t – 2.625)^2 + 240.25. This equation, $h(t)$ = –16(t – 2.625)^2 + 240.25, represents the function written in vertex form where the vertex is given by (2.625, 240.25). This means that the toy rocket reaches a maximum height of 240.25 meters 2.625 seconds after being fired from the top of the building";
- A correct answer in part B, as follows: The equation can be rewritten in a factored form as $h(t)$ = –2(2t – 13)(4t + 5). The toy rocket hits the ground 6.5 seconds after being fired from the top of the building;
- A correct explanation and work in part B, similar to "The function $h(t)$ = –16t^2 + 84t +130 represents the height of the toy rocket t seconds after being fired from the top of the building. To find the zeros of the function and therefore the time at which the rocket hits the ground, set $h(t)$ = 0 and factor –2 from each term to remove the negative factor and factor out the GCF as follows: 0 = –2(8t^2 – 42t – 65). Then factor the trinomial within the parentheses to give 0 = –2(2t – 13)(4t + 5). To solve for the zeros or t-intercepts, set each binomial equal to 0 to yield 2t – 13 = 0, which gives t = 13/2, AND 4t + 5 = 0, which gives t = –5/4. Since the time cannot be negative, only the positive intercept is used, which means that the rocket hits the ground 13/2 or 6.5 seconds after being fired from the top of the building."

3 The response demonstrates a strong understanding, but the work contains minor errors. A level 3 response is characterized by any two of the following:

- An answer for part A that is correct or contains one or two minor errors;
- Work and an explanation for part A that are incomplete or contain one or two minor errors;
- An answer for part B that is correct or contains one or two minor errors;
- Work and an explanation for part B that are incomplete or contain one or two minor errors.

2 The response demonstrates a basic but incomplete understanding. A level 2 response is characterized by all of the following:

- Work, an explanation, and an answer for part A that are basically correct but may be incomplete and include more than two minor errors;
- Work, an explanation, and an answer for part B that are basically correct but may be incomplete and include more than two minor errors.

1 The response demonstrates minimal understanding. A level 1 response is characterized by both of the following:

- Work, an explanation, and an answer for part A that are incomplete or contain one or more major errors but are not completely incorrect;
- Work, an explanation, and an answer for part B that are incomplete or contain one or more major errors but are not completely incorrect.

0 The response is completely incorrect, there is no response, or the response is off topic.

Standards:

CCSS.Math.Content.HSF-IF.C.8

Directions: Answer the following question(s).

4 The height in meters, $h(t)$, of a baseball t seconds after being thrown from the top of a hill is given by $h(t) = -4.9t^2 + 19.6t + 40.425$. Explain how to write $h(t)$ in vertex form. Then use this form of the equation to identify the maximum height attained by the ball and the time at which this height is reached. Show your work.

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Rubric: 2 Point(s)

2 The response is correct and complete. A sample 2-point response is shown below.

The equation can be rewritten as $h(t) = –4.9(t – 2)^2 + 60.025$ where the ball reaches a maximum height of 60.025 meters after 2 seconds.

Sample Correct Answer:

Take $h(t) = –4.9t^2 + 19.6t + 40.425$ and convert it to vertex form by completing the square. To complete the square, first factor the leading coefficient of –4.9 from the first two terms to give $h(t) = –4.9(t^2 – 4t) + 40.425$. Then, take half of the coefficient of the t-term within the parentheses and then square it to obtain $(–2)^2 = 4$. This value should be added within the parentheses and then subtracted from 40.425 outside the parentheses once it is multiplied by –4.9. This step yields $h(t) = –4.9(t^2 – 4t + 4) + 40.425 – 4.9(–4) = –4.9(t^2 – 4t + 4) + 40.425 + 19.6$ or $h(t) = –4.9(t^2 – 4t + 4) + 60.025$. Further factoring gives $h(t) = –4.9(t – 2)^2 + 60.025$. This equation, $h(t) = –4.9(t – 2)^2 + 60.025$, represents the function written in vertex form where the vertex is given by (2, 60.025). This means that the ball reaches a maximum height of 60.025 meters after 2 seconds of being released by the person throwing it.

1 The response is partially correct.

A response at this level includes work, an explanation, and an answer that are incomplete or that contain one or two minor errors.

0 The response is incorrect or there is no response.

Standards:

CCSS.Math.Content.HSF-IF.C.8